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Bifurcation of Periodic Solutions to Nonlinear Distributional Differential Equations

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This is a joint work with Carol Mesquita, Federal University of São Carlos, SP, Brazil.

We consider T-periodic problem for distributional (measure) differential equation

$$Dx = f(x,t) + g(\lambda, x, t).Du, \quad x(0) = x(T). \tag{P}$$

where $\Omega \subset \mathbb{R}^n$ and $\Lambda \subset \mathbb{R}$ are open, $f : \Omega \times [0,T] \to \mathbb{R}^n$, $g : \Lambda \times \Omega \times [0,T] \to \mathbb{R}^n$, $u\colon [0,T] \to \mathbb{R}$ is left-continuous on $(0,T]$ and has a bounded variation on $[0,T]$, Dx and Du are distributional derivatives of x and u, respectively, $g(\lambda, x, t).Du$ stands for the distributional product.

By a solution of (P) we mean a couple $(x, \lambda) \in BV([0,T], \mathbb{R}^n) \times \Lambda$ such that $x(0) = x(T)$, x is left-continuous on $(0,T]$, $x(t) \in \Omega$ for $t \in [0,T]$, the distributional product $g(x, \lambda, \cdot).Du$ makes a sense and the differential equality in (P) is satisfied in the distributional sense.

It is assumed that there exists $x_0 : [0,T] \to \mathbb{R}^n$ such that the pair (x_0, λ) is a solution of (P) for all $\lambda \in \Lambda$. Our goal is to state conditions necessary for that (x_0, λ_0) was the bifurcation point of problem (P).