Czech-Georgian Workshop on Boundary Value Problems
June 6-8, 2023, Brno, Czech Republic

Asymptotics of Nonlinear Fractional Differential Equations and Regular Variation

Pavel Řehák

Brno University of Technology, Brno, Czech Republic

rehak.pavel@fme.vutbr.cz

In the talk, we will consider the fractional differential equation of the form

$$\mathcal{D}^{\alpha+1}y = p(t)\Phi_\gamma(y),$$

$t \in [0,\infty)$, where $\Phi_\gamma(u) = |u|^\gamma \operatorname{sgn} u$, $\gamma \in (0,1)$ (the *sublinearity* condition), p is a continuous function on $[0,\infty)$, positive on $(0,\infty)$, and $\alpha \in (0,1)$; the fractional differential operator is of Caputo type. We will present (unimprovable) conditions guaranteeing that this equation possesses asymptotically superlinear solutions (i.e., the solutions y such that $\lim_{t\to\infty} y(t)/t = \infty$). We will show that all of these solutions are regularly varying and establish precise asymptotic formulae for them. (In the very special case, when the coefficient is asymptotically equivalent to a power function and the order of the equation is 2, our results reduce to the known theorems in their full generality.) In addition to the asymptotically superlinear solutions we will discuss also other (asymptotic) classes of solutions, some of them having no ODE analogy. We will reveal several substantial differences between the integer order and non-integer order case. We will highlight some of the tools used in the proofs such as theory of regular variation (in particular, the fractional Karamata integration theorem) and the fractional generalized l'Hospital which have the great potential to find applications in much wider "fractional" framework.