Lambert W Function and (Fractional) Delay Differential Equations

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The questions related to stability (and asymptotic properties) of the basic (test) first-order linear DDE can be treated by means of the Lambert W function. We show that this special (and multi-valued) function (despite its complex nature) can be manipulated, in some sense, in the real domain only. In particular, we can provide an easy "iff" condition ensuring that the real part of the principal branch is less than a given constant. Such a criterion then, very easily, enables to rediscover the stability region for the mentioned test equation. Morover, en extension to the fractional counterpart is straightforward, including a precise asymptotic description of the solutions.