Positive solutions to boundary value problems for systems of linear functional differential equations

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Consider the system of functional differential inequalities

$$\mathcal{D}\big(\sigma(t)\big)\big[u'(t)-\ell(u)(t)\big]\geq 0 \qquad \text{for a. e. } t\in[a,b], \qquad \varphi(u)\geq 0,$$

where $\ell : C\big([a,b];\mathbb{R}^n\big)\to L\big([a,b];\mathbb{R}^n\big)$ is a linear bounded operator, $\varphi : C\big([a,b];\mathbb{R}^n\big)\to\mathbb{R}^n$ is a linear bounded functional, $\sigma(t)=(\sigma_i(t))_{i=1}^n$ where $\sigma_i : [a,b]\to\{-1,1\}$ are measurable functions, and $\mathcal{D}\big(\sigma(t)\big)=\operatorname{diag}(\sigma_1(t),\dots,\sigma_n(t))$. We establish effective conditions guaranteeing that every absloutely continuous vector-valued function u satisfying the above-mentioned inequalities admits also the inequality $u(t)\geq 0$ for $t\in[a,b]$; in particular cases also the inequality $\mathcal{D}\big(\sigma(t)\big)u'(t)\geq 0$ for a. e. $t\in[a,b]$ is guaranteed.