p-Laplacian — maximum and comparison principles, uniqueness versus nonuniqueness

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We will present some recent results on maximum principles for the Cauchy problem for the parabolic *p*-Laplacian. In the first part of the talk we discuss the weak comparison principle for a parabolic *p*-Laplacian problem. While the weak comparison principle holds provided the reaction function $h(x, t, u)$ is nondecreasing and Lipschitz continuous with respect to u, we show that the weak comparison principle does not hold when the reaction function is not Lipschitz continuous even for x from an arbitrarily small part of the spatial domain. Namely, using the method of sub- and supersolutions we construct a nontrivial nonnegative weak solution to an initial-value problem which also possesses the trivial solution. This is a joint work with V. Bobkov, P. Girg, L. Kotrla and P. Takac. In the second part we consider continuous nonnegative solutions to a doubly nonlinear parabolic problem with the *p*-Laplacian with zero Dirichlet boundary conditions. For simplicity we assume that both the initial data and the reaction function are continuous and nonnegative and the reaction function does not depend on u. We show that for $1 < p < 2$ the speed of propagation is infinite in the sense that for any fixed time the solution is either everywhere positive or identically zero. In particular, if the initial data are nonzero at at least one point, then for small positive time the solution is positive in the whole domain, i.e., the strong maximum principle holds. This is a joint work with P. Girg, L. Kotrla and P. Takac.