Generalized linear differential equations in a Banach space (Kurzweil-Stieltjes integral and its applications)

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In the contribution we present new conditions ensuring the continuous dependence on a parameter k of solutions to linear integral equations of the form

$$x(t) = \widetilde{x}_k + \int_a^t \mathrm{d}[A_k]\, x + f_k(t) - f_k(a)\,, \quad t \in [a,b]\,, k \in \mathbb{N}\,,$$

where $-\infty < a < b < \infty$, X is a Banach space, $L(X)$ is the Banach space of linear bounded operators on X, $\widetilde{x}_k \in X$, $A_k : [a,b] \to L(X)$ have bounded variations on $[a,b]$, $f_k : [a,b] \to X$ are regulated on $[a,b]$. The integrals are understood as the abstract Kurzweil-Stieltjes integrals and the studied equations are usually called generalized linear differential equations (in the sense of J. Kurzweil, cf. [3] or [4]).

Our main theorem concerns the case when the variations $\operatorname{var}_a^b A_k$ need not be uniformly bounded and it is an analogy of the Opial's result [7] for ODEs. A crucial tool is the lemma that we call the Kiguradze lemma and which was taken from other context (see [1] and [2]) and which, to our opinion, somehow describes the common essential properties of linear operators generated by differential like equations.

Applications to linear dynamic equations on time scales are then enabled by their relationship with generalized differential equations as disclosed by A. Slavík in [8].

References

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